Generalized lagrangian of the Rarita–Schwinger field

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We derive the most general lagrangian of the free massive Rarita–Schwinger field, which generalizes the previously known ones. The special role of the reparameterization transformation is discussed.

I. INTRODUCTION

The Rarita-Schwinger (R.-S.) vector-spinor field $\Psi^\mu(x)$ is used for description of the spin-3/2 particles in QFT [1]. But this field has too many degrees of freedom for spin-3/2 and contains in fact also two spin-1/2 representations. These "extra" spin-1/2 components are usually supposed to be unphysical, but the control of this property becomes not so evident after inclusion of interactions.

In this paper we derive the most general form of the R.-S. lagrangian without additional assumption about the nature of spin-1/2 components. Depending on the choice of parameters the $s = 1/2$ components may have the poles in the complex energy plane or not. The purpose of such construction is twofold. First of all, it may be useful at renormalization of the dressed R.-S. field propagator even if we require the spin-1/2 components to be unphysical. Another aim may be related with properties of the hypothetical R.-S. multiplet, corresponding to Ψ^μ field and consisting of particles $J^P = 3/2^+, 1/2^+, 1/2^-$.

There exist some examples of generalized lagrangians [2, 3, 4] which are constructed by different methods (usually with using of artificial methods) and for different purposes. In contrast to previous works our deriving of the generalized R.-S. lagrangian is a straightforward: we investigate the propagator instead of equations of motion and utilize different bases [5] to control its properties. Such approach is more transparent at any step but needs some technical details, which are collected in Appendix.

II. LAGRANGIAN

Free lagrangian of the Rarita-Schwinger field is defined by differential operator $S^{\mu\nu}$, which is in fact the inverse propagator

$$\mathscr{L} = \overline{\Psi}^\mu S_{\mu\nu} \Psi^\nu. \tag{1}$$

Let us recall the standard choice [6] for $S^{\mu\nu}$:

$$S^{\mu\nu} = (\hat{p} - M) g^{\mu\nu} + A(\gamma^\mu p^\nu + \gamma^\nu p^\mu) + \frac{1}{2}(3A^2 + 2A + 1)\gamma^\mu \hat{p} \gamma^\nu + M(3A^2 + 3A + 1)\gamma^\mu \gamma^\nu. \tag{2}$$

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Here M is the mass of R.-S. field, $p_\mu = i\partial_\mu$ and A is an arbitrary real parameter. Equations of motion, following from (2), lead to constrains $p\Psi = \gamma\Psi = 0$, and therefore to exclusion of the spin-1/2 degrees of freedom. In other words, the corresponding terms in propagator should not have poles in energy.

Let us formulate the main requirements for lagrangian:

1. The fermion lagrangian is linear in derivatives.
2. It should be hermitian $\mathscr{L}^\dagger = \mathscr{L}$ or $\gamma^0(S^{\mu\nu})^\dagger\gamma^0 = S^{\nu\mu}$.
3. The spin-3/2 contribution has standard pole form (to be specified below).
4. Lagrangian should not be singular at $p \to 0$. This point is rather evident but it happens that some rough methods generate singularities in a propagator, see, e.g. discussion in [7].

The suitable starting point to construct the generalized lagrangian is the most general decomposition of $S^{\mu\nu}$ in γ-matrix basis[1] (A1). The first requirement remains 6 complex coefficients in (A1)

$$\begin{aligned} S^{\mu\nu} &= g^{\mu\nu}\cdot s_1 + \hat{p}g^{\mu\nu}\cdot s_4 + p^\mu\gamma^\nu\cdot s_5 + \gamma^\mu p^\nu \cdot s_6 + \sigma^{\mu\nu}\cdot s_7 + i\epsilon^{\mu\nu\lambda\rho}\gamma_\lambda\gamma^5 p_\rho\cdot s_{10} = \\ &= g^{\mu\nu}(s_1 - s_7) + \hat{p}g^{\mu\nu}(s_4 - s_{10}) + p^\mu\gamma^\nu(s_5 + s_{10}) + \gamma^\mu p^\nu(s_6 + s_{10}) + \gamma^\mu\gamma^\nu s_7 - \gamma^\mu\hat{p}\gamma^\nu s_{10}. \end{aligned} \tag{3}$$

If we start from the γ-matrix decomposition with nonsingular coefficients the fourth requirement is fulfilled automatically.

This expression satisfies the condition $\gamma^0(S^{\mu\nu})^\dagger\gamma^0 = S^{\nu\mu}$, if s_1, s_4, s_7, s_{10} are real parameters while s_5 and $s_6 = s_5^*$ may be complex. It is convenient to introduce the new notations

$$s_1 = r_1, \quad s_4 = r_4, \quad s_7 = r_7, \quad s_{10} = r_{10}, \quad s_5 = r_5 + ia_5, \quad s_6 = r_5 - ia_5,$$

where all parameters are real.

To take into account the third requirement, we need to recognize the spin-3/2 part of inverse propagator. It is easy to do in the $\hat{p}$-basis (see Appendix A for details)

$$S^{\mu\nu} = (\hat{p} - M)\left(\mathcal{P}^{3/2}\right)^{\mu\nu} + (\text{spin-1/2 contributions}). \tag{4}$$

Reversing the Eq.(4), we obtain propagator with the standard pole behavior of spin-3/2 contribution

$$G^{\mu\nu} = \frac{1}{\hat{p} - M}\left(\mathcal{P}^{3/2}\right)^{\mu\nu} + (\text{spin-1/2 contributions}). \tag{5}$$

Eq.(4) gives[2] (using formulae (A9), (A10) for transition from one basis to another)

$$S_1 = s_1 - s_7 = -M, \quad S_2 = s_4 - s_{10} = 1. \tag{6}$$

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[1] We use conventions of Bjorken and Drell textbook [8]: $\varepsilon_{0123} = 1$, $\gamma^5 = \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ except that $\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$.

[2] We need to use different bases, so to distinguish them we use different notations: $s_i, S_i, \bar{S}_i$ for coefficients in γ-, $\hat{p}$- and Λ-basis respectively.

So the r_7, r_{10} are dependent values

$$r_7 = M + r_1, \quad r_{10} = r_4 - 1$$

and we come to four-parameter (r_1, r_4, r_5, a_5) lagrangian, which satisfies all the necessary requirements.

$$S^{\mu\nu} = g^{\mu\nu}(\hat{p}-M)+p^\mu\gamma^\nu(r_5+r_4-1+ia_5)+p^\nu\gamma^\mu(r_5+r_4-1-ia_5)+\gamma^\mu\gamma^\nu(M+r_1)-\gamma^\mu\hat{p}\gamma^\nu(r_4-1). \tag{7}$$

Other bases may be useful, so let us write down the corresponding decomposition coefficients (here $E = \sqrt{p^2}$):

γ-basis:	$\hat{p}$-basis:	Λ-basis:
$s_1 = r_1,$	$S_1 = -M,$	$\bar{S}_1 = -M + E,$
$s_2 = s_3 = 0,$	$S_2 = 1,$	$\bar{S}_2 = -M - E,$
$s_4 = r_4,$	$S_3 = 2M + 3r_1,$	$\bar{S}_3 = (2M + 3r_1) + E(3r_4 - 2),$
$s_5 = r_5 + ia_5,$	$S_4 = 3r_4 - 2,$	$\bar{S}_4 = (2M + 3r_1) - E(3r_4 - 2),$
$s_6 = r_5 - ia_5,$	$S_5 = r_1,$	$\bar{S}_5 = r_1 + E(2r_5 + r_4),$
$s_7 = M + r_1,$	$S_6 = r_4 + 2r_5,$	$\bar{S}_6 = r_1 - E(2r_5 + r_4),$
$s_8 = s_9 = 0,$	$S_7 = \sqrt{3}\ E(r_5 - ia_5),$	$\bar{S}_7 = \sqrt{3}\ \big[-(M + r_1) + E(r_5 - ia_5)\big],$
$s_{10} = r_4 - 1,$	$S_8 = -\sqrt{3}\ (M + r_1)/E,$	$\bar{S}_8 = \sqrt{3}\ \big[(M + r_1) + E(r_5 - ia_5)\big],$
	$S_9 = \sqrt{3}\ E(r_5 + ia_5),$	$\bar{S}_9 = \sqrt{3}\ \big[(M + r_1) + E(r_5 + ia_5)\big],$
	$S_{10} = \sqrt{3}\ (M + r_1)/E,$	$\bar{S}_{10} = \sqrt{3}\ \big[-(M + r_1) + E(r_5 + ia_5)\big].$

III. PROPAGATOR OF THE RARITA-SCHWINGER FIELD

To build the propagator of the Rarita-Schwinger field we need to reverse (7)

$$G^{\mu\nu}(p) = \left(S^{-1}\right)^{\mu\nu}, \tag{8}$$

and the Λ-basis is convenient here. Reversing of the spin-tensor leads to set of equations for the scalar coefficients $\bar{G}_i$ [5, 9].

$$\begin{aligned}
\bar{G}_1\bar{S}_1 &= 1, \\
\bar{G}_2\bar{S}_2 &= 1, \\
\bar{G}_3\bar{S}_3 + \bar{G}_7\bar{S}_{10} &= 1, \qquad & \bar{G}_4\bar{S}_4 + \bar{G}_8\bar{S}_9 &= 1, \\
\bar{G}_3\bar{S}_7 + \bar{G}_7\bar{S}_6 &= 0, \qquad & \bar{G}_4\bar{S}_8 + \bar{G}_8\bar{S}_5 &= 0, \\
\bar{G}_5\bar{S}_5 + \bar{G}_9\bar{S}_8 &= 1, \qquad & \bar{G}_6\bar{S}_6 + \bar{G}_{10}\bar{S}_7 &= 1, \\
\bar{G}_5\bar{S}_9 + \bar{G}_9\bar{S}_4 &= 0, \qquad & \bar{G}_6\bar{S}_{10} + \bar{G}_{10}\bar{S}_3 &= 0.
\end{aligned} \tag{9}$$

The equations are easy to solve:

$$\begin{aligned}
&\bar{G}_1 = \frac{1}{\bar{S}_1}, \quad \bar{G}_2 = \frac{1}{\bar{S}_2}, \\
&\bar{G}_3 = \frac{\bar{S}_6}{\Delta_1}, \quad \bar{G}_4 = \frac{\bar{S}_5}{\Delta_2}, \quad \bar{G}_5 = \frac{\bar{S}_4}{\Delta_2}, \quad \bar{G}_6 = \frac{\bar{S}_3}{\Delta_1}, \\
&\bar{G}_7 = \frac{-\bar{S}_7}{\Delta_1}, \quad \bar{G}_8 = \frac{-\bar{S}_8}{\Delta_2}, \quad \bar{G}_9 = \frac{-\bar{S}_9}{\Delta_2}, \quad \bar{G}_{10} = \frac{-\bar{S}_{10}}{\Delta_1},
\end{aligned} \tag{10}$$

where

$$\Delta_1 = \bar{S}_3\bar{S}_6 - \bar{S}_7\bar{S}_{10}, \qquad \Delta_2 = \bar{S}_4\bar{S}_5 - \bar{S}_8\bar{S}_9. \tag{11}$$

R.-S. propagator in the spin-3/2 sector ($\bar{G}_1, \bar{G}_2$ coefficients) is similar to usual Dirac propagator. As for spin-1/2 sector ($\bar{G}_3 - \bar{G}_{10}$), it looks like mixing of two bare propagators with non-diagonal transitions. Zeros of denominators Δ_1, Δ_2 are the poles of R.-S. propagator with quantum numbers $s = 1/2$.

Let us write down the denominators following from our lagrangian (7):

$$\Delta_1(E) = -M(3M + 4r_1) + 2E(Mr_5 - Mr_4 - r_1) + E^2(-3a_5^2 - 3(r_4 + r_5)^2 + 4r_5 + 2r_4),$$
$$\Delta_2(E) = \Delta_1(E \to -E).$$

If we want the spin-1/2 contributions to be unphysical, it requires $\Delta_1 = const$ and we come to conditions:

$$\begin{aligned} M(r_5 - r_4) - r_1 = 0, \\ 3a_5^2 + 3(r_4 + r_5)^2 - 4r_5 - 2r_4 = 0. \end{aligned} \tag{12}$$

One can rewrite it in terms of sum and difference $\sigma = r_5 + r_4$, $\delta = r_5 - r_4$:

$$\begin{aligned} r_1 &= M\delta, \\ \delta &= 3(\sigma^2 - \sigma + a_5^2). \end{aligned} \tag{13}$$

Let us consider some particular cases of our lagrangian (7)

- To obtain unphysical spin-1/2 sector we should require the conditions (12). If these relations are fulfilled, we can return to the standard R.-S. lagrangian (2), if to put $a_5 = 0$ and denote $\sigma = A + 1$.

- Generalization of the standard lagrangian (2) was suggested by Pilling [4]. His lagrangian corresponds to unphysical $s = 1/2$ sector, has two parameters and may be obtained (in $d = 4$ space) from our lagrangian (7) after imposing of the conditions (12) and changing of notations. Note that the procedure of deriving [4] is used some trick [10] and looks not too transparent.

- Lagrangian suggested by Kirchbach and Napsuciale [3]

  $$S^{\mu\nu} = i\varepsilon^{\mu\nu\lambda\rho}\gamma^5\gamma_\lambda p_\rho - Mg^{\mu\nu}$$

  corresponds to the choice

  $$r_4 = r_5 = a_5 = 0, \qquad r_1 = -M.$$

  It leads to poles in $s = 1/2$ sector

  $$\Delta_1 = M(M + 2E), \quad \Delta_1 = M(M - 2E)$$

  and presence of these poles contradicts to further analysis [3] of this lagrangian.

IV. REPARAMETRIZATION

There exists the well-known transformation of R.-S. field

$$\Psi_\mu \to \Psi'_\mu : \quad \Psi_\mu = \theta_{\mu\nu}(B)\Psi'^\nu, \tag{14}$$

where $\theta_{\mu\nu}(B) = g_{\mu\nu} + B\gamma_\mu\gamma_\nu$ and $B = b + i\beta$ is a complex parameter.

This transformation doesn't touch the spin-3/2 because $(\mathcal{P}^{3/2})^{\mu\nu}$ operator is orthogonal to γ^α. So if to apply it to our inverse propagator (7)

$$S_{\mu\nu} \to S'_{\mu\nu} = \theta_{\mu\alpha}(B^*)S^{\alpha\beta}\theta_{\beta\nu}(B), \tag{15}$$

one can see that $S'_{\mu\nu}$ keeps all the properties of $S_{\mu\nu}$ (7). It means that in fact we have reparametrization – after transformation of the (7) we obtain the same operator with changed parameters

$$\theta_{\mu\alpha}(B^*)S^{\alpha\beta}(r_1, r_4, r_5, a_5)\theta_{\beta\nu}(B) = S_{\mu\nu}(r_1', r_4', r_5', a_5'). \tag{16}$$

Direct calculations confirm it, the transformed parameters are the following:

$$\begin{aligned}
r_1' &= r_1 + 2(3M + 4r_1)(2b^2 + b + \beta^2),\\
a_5' &= a_5(1 + 4b) + 2\beta(2r_4 + 2r_5 - 1),\\
r_4' &= r_4 + 2b^2(4r_4 - 4r_5 - 3) + 2b(3r_4 - r_5 - 2) + 2\beta^2(4r_4 - 4r_5 - 3) - 2\beta a_5,\\
r_5' &= r_5 - 2b^2(4r_4 - 4r_5 - 3) - 2b(r_4 - 3r_5 - 1) - 2\beta^2(4r_4 - 4r_5 - 3) - 2\beta a_5,
\end{aligned} \tag{17}$$

or, for sum and difference ($\sigma = r_5 + r_4$, $\delta = r_5 - r_4$):

$$\begin{aligned}
\sigma' &= \sigma + 2b(2\sigma - 1) - 4\beta a_5,\\
\delta' &= \delta + 2(3 + 4\delta)(2b^2 + b + 2\beta^2),\\
a_5' &= a_5(1 + 4b) + 2\beta(2\sigma - 1).
\end{aligned}$$

Not so evident but important property of the θ-transformation is that it doesn't change the pole positions (masses) of spin-1/2 representations. It may be seen from the transformation law of denominators Δ_1, Δ_2 (11) of the propagator:

$$\Delta_i(E) \to \Delta_i'(E) = \Delta_i(E) \cdot \mid 1 + 4B \mid^2 . \tag{18}$$

It means that not all parameters are essential for spectrum of $s = 1/2$ and this fact may be useful for simplification. Applying the θ-transformation to our $S^{\mu\nu}$ operator (16) we can eliminate two of four parameters

$$\sigma' = 0, \qquad a_5' = 0.$$

After some bit of algebra one can find the transformation parameters

$$2b = -\frac{\sigma(2\sigma - 1) + 2a_5^2}{(2\sigma - 1)^2 + a_5^2}, \qquad 2\beta = \frac{a_5}{(2\sigma - 1)^2 + a_5^2},$$

under the condition

$$(2\sigma - 1)^2 + a_5^2 \neq 0.$$

Then it is useful to renormalize the R.-S. field[3]

$$\mathscr{L} = \overline{\Psi}^{\mu} S_{\mu\nu} \Psi^{\nu} = \overline{\Psi}^{\mu} \theta_{\mu\alpha}(\bar{B}^*) \cdot \theta^{\alpha\beta}(B^*) S_{\beta\rho} \theta^{\rho\lambda}(B) \cdot \theta_{\lambda\nu}(\bar{B}) \Psi^{\nu} = \overline{\Psi}'^{\alpha} S_{\prime\alpha\lambda} \Psi'^{\lambda}, \tag{19}$$

where $S'^{\alpha\beta}$ contains only two parameters

$$S'^{\alpha\beta} = S^{\alpha\beta}(r_1', \delta', \sigma' = 0, a_5' = 0)$$

and renormalized field is

$$\Psi'^{\lambda} = \theta^{\lambda\nu}(\bar{B}) \Psi_{\nu}. \tag{20}$$

After field renormalization (20) we have two-parameter free lagrangian, but now the θ-factor will appear in the interaction lagrangian through (14). Such θ-factor with arbitrary parameter (so called "off-shell" parameter) traditionally exists in interaction lagrangian. Nevertheless, the meaning of this parameter and its choice is rather controversial, see, e.g. discussion in [11].

V. CONCLUSION

We obtained the four-parametric lagrangian which satisfies all general requirements and generalizes all known lagrangians for Rarita-Schwinger field. We used a straightforward procedure for its deriving which utilize different bases for spin-tensor $S^{\mu\nu}(p)$ and studying of propagator instead of equation of motion. The corresponding propagator has standard form of the spin-3/2 contribution. As for spin-1/2 terms, they can have poles in the energy plane which positions and residues depends on the parameters.

We found that the θ-transformation plays some special role in our lagrangian: this is a reparametrization which doesn't move the spin-1/2 poles. We suppose that the meaning of this degree of freedom may be more clear in case of physical spin-1/2 sector.

We suppose that investigation of the phenomenology of Rarita-Schwinger multiplet may be interesting and present work is a necessary step in this direction.

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APPENDIX A: DECOMPOSITION OF SPIN-TENSOR

Propagator or self-energy of the R.-S. field has two spinor and two vector indices and depends on momentum p. We will denote such object as $S^{\mu\nu}(p)$, omitting spinor indices, and will call it shortly as a spin-tensor. In our consideration we need to use different bases for this object.

1. Most evident is a γ-matrix decomposition. It's easy to write down all possible γ-matrix structures with two vector indices. Altogether there are 10 terms in decomposition of spin-tensor, if parity is conserved.

$$\begin{aligned} S^{\mu\nu}(p) =& g^{\mu\nu} \cdot s_1 + p^{\mu} p^{\nu} \cdot s_2 + \hat{p} p^{\mu} p^{\nu} \cdot s_3 + \hat{p} g^{\mu\nu} \cdot s_4 + p^{\mu} \gamma^{\nu} \cdot s_5 + \gamma^{\mu} p^{\nu} \cdot s_6 + \\ &+ \sigma^{\mu\nu} \cdot s_7 + \sigma^{\mu\lambda} p_{\lambda} p^{\nu} \cdot s_8 + \sigma^{\nu\lambda} p_{\lambda} p^{\mu} \cdot s_9 + \gamma_{\lambda} \gamma^5 \imath \varepsilon^{\lambda\mu\nu\rho} p_{\rho} \cdot s_{10}. \end{aligned} \tag{A1}$$

[3] Recall that $\theta(a)\theta(b) = \theta(a + b + 4ab)$, so inverse transformation $\theta(a)\theta(\bar{a}) = I$ is defined by parameter $\bar{a} = -a/(1 + 4a)$.

Here $s_i(p^2)$ are the Lorentz-invariant coefficients and $\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$.

This is a good starting point of any consideration, since this basis is complete, non-singular and free of kinematical constrains. But this basis is not convenient at multiplication (e.g. in Dyson summation) because elements of basis are not orthogonal to each other.

2. There is another basis (e.g. [12]) for $S^{\mu\nu}$, which we call as $\hat{p}$-basis. Decomposition of any spin-tensor in this basis has the form

$$\begin{aligned} S^{\mu\nu}(p) \;=\; & (S_1 + \hat{p}S_2)\big(\mathcal{P}^{3/2}\big)^{\mu\nu} + (S_3 + \hat{p}S_4)\big(\mathcal{P}^{1/2}_{11}\big)^{\mu\nu} + (S_5 + \hat{p}S_6)\big(\mathcal{P}^{1/2}_{22}\big)^{\mu\nu} + \\ & + (S_7 + \hat{p}S_8)\big(\mathcal{P}^{1/2}_{21}\big)^{\mu\nu} + (S_9 + \hat{p}S_{10})\big(\mathcal{P}^{1/2}_{12}\big)^{\mu\nu}, \end{aligned} \tag{A2}$$

where appeared the well-known tensor operator [7, 12, 13]

$$\begin{aligned} \big(\mathcal{P}^{3/2}\big)^{\mu\nu} &= g^{\mu\nu} - \big(\mathcal{P}^{1/2}_{11}\big)^{\mu\nu} - \big(\mathcal{P}^{1/2}_{22}\big)^{\mu\nu}, \\ \big(\mathcal{P}^{1/2}_{11}\big)^{\mu\nu} &= 3\pi^\mu\pi^\nu, \\ \big(\mathcal{P}^{1/2}_{22}\big)^{\mu\nu} &= \frac{p^\mu p^\nu}{p^2}, \\ \big(\mathcal{P}^{1/2}_{21}\big)^{\mu\nu} &= \sqrt{\frac{3}{p^2}}\cdot \pi^\mu p^\nu, \\ \big(\mathcal{P}^{1/2}_{12}\big)^{\mu\nu} &= \sqrt{\frac{3}{p^2}}\cdot p^\mu \pi^\nu, \end{aligned} \tag{A3}$$

which are written here in a non-standard form. Here we introduced the "vector"

$$\pi^\mu = \frac{1}{3p^2}(-p^\mu + \gamma^\mu\hat{p})\hat{p} \tag{A4}$$

with the following properties:

$$(\pi p) = 0, \quad (\gamma\pi) = (\pi\gamma) = 1, \quad (\pi\pi) = \frac{1}{3}, \quad \hat{p}\pi^\mu = -\pi^\mu\hat{p}. \tag{A5}$$

3. The most convenient at multiplication basis is build by combining the $\mathcal{P}^i_{\mu\nu}$ operators (A3) and the off-shell projection operators $\Lambda^\pm$

$$\Lambda^\pm = \frac{1}{2}\left(1 \pm \frac{\hat{p}}{\sqrt{p^2}}\right), \tag{A6}$$

where we assume $\sqrt{p^2}$ to be the first branch of analytical function. Ten elements of this basis look as

$$\begin{aligned} &\mathcal{P}_1 = \Lambda^+\mathcal{P}^{3/2}, \quad \mathcal{P}_3 = \Lambda^+\mathcal{P}^{1/2}_{11}, \quad \mathcal{P}_5 = \Lambda^+\mathcal{P}^{1/2}_{22}, \quad \mathcal{P}_7 = \Lambda^+\mathcal{P}^{1/2}_{21}, \quad \mathcal{P}_9 = \Lambda^+\mathcal{P}^{1/2}_{12}, \\ &\mathcal{P}_2 = \Lambda^-\mathcal{P}^{3/2}, \quad \mathcal{P}_4 = \Lambda^-\mathcal{P}^{1/2}_{11}, \quad \mathcal{P}_6 = \Lambda^-\mathcal{P}^{1/2}_{22}, \quad \mathcal{P}_8 = \Lambda^-\mathcal{P}^{1/2}_{21}, \quad \mathcal{P}_{10} = \Lambda^-\mathcal{P}^{1/2}_{12}, \end{aligned} \tag{A7}$$

where tensor indices are omitted. Decomposition in this basis:

$$S^{\mu\nu}(p) = \sum_{A=1}^{10} \bar{S}_A \mathcal{P}^{\mu\nu}_A. \tag{A8}$$

Coefficients of $S^{\mu\nu}$ in $\hat{p}$- and γ-bases are related by

$$\begin{aligned}
s_1 &= \frac{1}{3}(2S_1 + S_3), & s_2 &= \frac{1}{3p^2}(-2S_1 - S_3 + 3S_5),\\
s_3 &= \frac{1}{3p^2}\Big(- 2S_2 - S_4 + 3S_6 - \sqrt{\frac{3}{p^2}}(S_7 + S_9)\Big), & s_4 &= \frac{1}{3}(2S_2 + S_4),\\
s_5 &= \frac{1}{\sqrt{3p^2}}S_9, & s_6 &= \frac{1}{\sqrt{3p^2}}S_7,\\
s_7 &= \frac{1}{3}(-S_1 + S_3), & s_8 &= \frac{1}{3p^2}\big(S_1 - S_3 - \sqrt{3p^2}S_8\big),\\
s_9 &= \frac{1}{3p^2}\big(- S_1 + S_3 - \sqrt{3p^2}S_{10}\big), & s_{10} &= \frac{1}{3}(-S_2 + S_4).
\end{aligned} \tag{A9}$$

Reversed relations:

$$\begin{aligned}
S_1 &= s_1 - s_7, & S_2 &= s_4 - s_{10},\\
S_3 &= s_1 + 2s_7, & S_4 &= s_4 + 2s_{10},\\
S_5 &= s_1 + p^2 s_2, & S_6 &= p^2 s_3 + s_4 + s_5 + s_6,\\
S_7 &= \sqrt{3p^2}s_6, & S_8 &= -\sqrt{\frac{3}{p^2}}(s_7 + p^2 s_8),\\
S_9 &= \sqrt{3p^2}s_6, & S_{10} &= \sqrt{\frac{3}{p^2}}(s_7 + p^2 s_8).
\end{aligned} \tag{A10}$$

Transition from $\hat{p}$- to Λ-basis:

$$\begin{aligned}
\bar{S}_1 &= S_1 + ES_2, & \bar{S}_3 &= S_3 + ES_4, & \bar{S}_5 &= S_5 + ES_6, & \bar{S}_7 &= S_7 + ES_8, & \bar{S}_9 &= S_9 + ES_{10},\\
\bar{S}_2 &= S_1 - ES_2, & \bar{S}_4 &= S_3 - ES_4, & \bar{S}_6 &= S_5 - ES_6, & \bar{S}_8 &= S_7 - ES_8, & \bar{S}_{10} &= S_9 - ES_{10}.
\end{aligned} \tag{A11}$$

Let us note that $\hat{p}$- and Λ-bases are singular at point $p^2 = 0$. As for branch point $\sqrt{p^2}$ appearing in different terms, it is canceled in total expression. But poles $1/p^2$ don't cancel automatically, if we work in $\hat{p}$- or Λ-basis. First of all, one can see from (A10) that the $S_7 - S_7$ should have kinematical $\sqrt{p^2}$ factors:

$$S_7 = \sqrt{p^2}\tilde{S}_7, \quad S_8 = \frac{1}{\sqrt{p^2}}\tilde{S}_7, \quad S_9 = \sqrt{p^2}\tilde{S}_9, \quad S_{10} = \frac{1}{\sqrt{p^2}}\tilde{S}_{10}, \tag{A12}$$

and $\tilde{S}_i$ don't have branch point at origin. After it we see from (A9) conditions of absence of $1/p^2$ poles:

$$\begin{aligned}
2S_1(0) + S_3(0) - 3S_5(0) &= 0,\\
S_1(0) - S_3(0) - \sqrt{3}\tilde{S}_8(0) &= 0,\\
2S_2(0) + S_4(0) - 3S_6(0) - \sqrt{3}(\tilde{S}_7(0) + \tilde{S}_9(0)) &= 0,\\
\tilde{S}_8(0) - \tilde{S}_{10}(0) &= 0.
\end{aligned} \tag{A13}$$

APPENDIX B: EQUATIONS OF MOTION

Usually the equations of motion are used for analysis of Rarita-Schwinger field content. In this approach the "extra" spin-1/2 components are excluded by additional conditions

$p_\mu \Psi^\mu = 0$, $\gamma_\mu \Psi^\mu = 0$ which should be the consequences of equations of motion. Let us analyze the equations of motion, which result from our four-parameter lagrangian (7).

Contracting the equations of motion with p_μ and γ_μ one can get

$$\begin{aligned} p_\mu S^{\mu\nu}\Psi_\nu = 0, \\ \gamma_\mu S^{\mu\nu}\Psi_\nu = 0. \end{aligned}$$

Our lagrangian (7) leads to the following secondary constrains:

$$\begin{aligned} &\Big[(r_4 + r_5 - ia_5)\hat{p} - M\Big](p\Psi) + \Big[p^2(r_5 + ia_5) + (M + r_1)\hat{p}\Big](\gamma\Psi) = 0, \\ 2&\Big[2r_4 + 2r_5 - 1 + 2ia_5\Big](p\Psi) + \Big[(3M + 4r_1) + (-3r_4 + r_5 + 2 + ia_5)\hat{p}\Big](\gamma\Psi) = 0. \end{aligned} \tag{B1}$$

This is a system of linear equations (with matrix coefficients) in $(p\Psi)$, $(\gamma\Psi)$. One can find conditions when this system has only trivial solution $(p\Psi) = (\gamma\Psi) = 0$. Let us express $(p\Psi)$ from the second equation and substitute to the first. It gives the equation in $(\gamma\Psi)$:

$$\Delta(p)(\gamma\Psi) = 0, \tag{B2}$$

where $\Delta(p)$ is operator of the form

$$\Delta(p) = -M(3M + 4r_1) + \hat{p}\cdot 2\Big(r_1 + Mr_4 - Mr_5\Big) + p^2\Big(-3(r_4 + r_5)^2 - 3a_5^2 + 2r_4 + 4r_5\Big).$$

Eq. (B2) is in fact some wave equation in the spinor $(\gamma\Psi)$. This equation has only trivial solution if $\Delta(p) = const$, or

$$\begin{aligned} r_1 + Mr_4 - Mr_5 = 0, \\ 3a_5^2 + 3(r_4 + r_5)^2 - 2r_4 - 4r_5 = 0. \end{aligned} \tag{B3}$$

These relations provide the constrains $(p\Psi) = 0$, $(\gamma\Psi) = 0$ and the we can see that the same conditions (12) guarantee the absence of poles $s = 1/2$.

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[1] W. Rarita and J. Schwinger. *Phys. Rev.* **60** (1941) 61.

[2] H. Haberzettl. nucl-th/9812043.

[3] M. Kirchbach and M. Napsuciale. hep-ph/0407179.

[4] T. Pilling. *Int.J.Mod.Phys.* **A20** (2005) 2715.

[5] A. E. Kaloshin and V. P. Lomov. *Mod.Phys.Lett.* **A19** (2004) 135.

[6] P. A. Moldauler and K. M. Case. *Phys. Rev.* **102** (1956) 102.

[7] M. Benmerrouche, R. M. Davidson and N. C. Mukhopadhyay. *Phys. Rev.* **C39** (1989) 2339.

[8] J.D.Bjorken and S.D.Drell. Relativistic quantun mechanics. McGraw-Hill Book Company, 1964.

[9] A. E. Kaloshin and V. P. Lomov. hep-ph/0409052

[10] A. Aurilia, H. Umezava. *Phys. Rev.* **182** (1969) 1686.

[11] L. M. Nath,B. Etemadi and J. D. Kimel. *Phys. Rev.* **D3** (1971) 2153.

[12] V. Pascalutsa and O. Scholten. *Nuc. Phys.* **A591** (1995) 658.

[13] P. van Nieuwenhuizen. *Phys. Rep.* **68** (1981) 189.